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SINGAPORE'S LIFE PROGRAM: ACTUARIAL FRAMEWORK, LONGEVITY RISK AND IMPACT OF ANNUITY FUND RETURN

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The Central Provident Fund (CPF) is a defined-contribution savings plan forming the key pillar of the pension system in Singapore. The CPF Lifelong Income For the Elderly (LIFE) program, which provides lifetime income for retirees, is a mandatory pension scheme for all Singapore residents. In this paper we construct an actuarial framework to analyze the LIFE program. We use this framework to study the plan payout outcomes with respect to changes in mortality and annuity fund return assumptions. We also examine the effects of some possible changes in the program on the payouts and bequests.

Keywords: Life annuity; inflation risk; investment returns; longevity risk.

JEL Classification: G22, G32

1. Introduction

The Singapore Central Provident Fund (CPF) Board introduced the Lifelong Income For the Elderly (LIFE) scheme in 2009 to provide a deferred whole-life annuity to retirees with possible refund of premium as a bequest upon death. All CPF members aged 55 or above with sufficient funds in their CPF accounts are required to join the scheme.[1] Currently, the LIFE scheme consists of two plans: the Standard and Basic plans. The Standard Plan provides more monthly payouts and less bequests, while the Basic Plan provides less monthly payouts and more bequests. Other than this difference, not much is known about the sensitivity of the payouts and bequests with respect to mortality and interest rate assumptions, as well as other possible plan changes such as step-up of payouts and deferment of annuity.

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[1] All working citizens and permanent residents in Singapore are required to open CPF accounts. Some members are exempted from the LIFE scheme due to medical conditions or equivalent possession of private pensions.

The LIFE scheme is a unique pension program combining life insurance elements and retirement financing, which is not commonly found in other pension plans or life annuity products. In this paper, we construct an actuarial framework to analyze the LIFE scheme. Based on the actuarial equivalence principle, we construct algorithms to project the payouts and bequests of the scheme. We then use this platform to study the sensitivity of the plan cashflows with respect to changes in model assumptions and parameters. Our framework can be adopted to analyze the effects of possible changes in scheme features on the payouts and bequests. In particular, we consider the effects of deferment of the monthly payouts and payout streams with step-ups. To the best of our knowledge, this paper is the first attempt in the academic literature to analyze the LIFE scheme actuarially.[2]

The remainder of this paper is organized as follows. Section 2 describes the general features of the CPF LIFE scheme. Section 3 outlines our actuarial models, which are used to derive the monthly payouts and bequests. Section 4 discusses the model assumptions for the plans and provides some cashflow examples for illustration. Section 5 analyzes the sensitivity of the monthly payouts to the model assumptions. Section 6 evaluates the impacts of two possible changes in the LIFE scheme: plans with annual step-up in payouts and deferment of start age of the payouts. Finally, Section 7 summarizes our findings and discusses future research directions.

2. The CPF LIFE Scheme

CPF is a comprehensive social security savings system that enables working Singapore citizens and permanent residents to set aside funds for retirement. Both employees and employers are required to make monthly CPF contributions, which go into three accounts: Ordinary Account (OA), Special Account (SA) and Medisave Account (MA). Each account serves different purposes that cater to members' needs.[3]

Under the CPF LIFE scheme, when a CPF member reaches age 55, some savings in the OA and SA are transferred to a newly created Retirement Account (RA). The member is also required to choose between two whole-life annuity plans: the Standard and Basic plans.[4] Annuity Funds (AFs) will then be created by using contributions from the RA. The amounts to be transferred to the AFs are determined according to the plan selected.[5] From then onward, the CPF LIFE scheme provides the CPF member with a monthly payout starting from the Draw Down Age (DDA), which is set at 65 prior to 2016, for as long as the member lives. When the member dies, a bequest equal to the balance in the member's RA plus the difference between the amount of annuity premiums contributed and the amount of payouts received from the AFs is paid to the member's beneficiaries. If the

[2] We will focus our analysis on the LIFE scheme features as applicable in 2015. The CPF Board has introduced some changes to the scheme for members joining after 2016. These changes, however, can be easily incorporated into our framework.

[3] See Fong *et al.* (2011) for a comprehensive survey of Singapore's pension system and the uses of the different CPF accounts.

[4] If no plan is chosen within six months, the member is automatically placed on the default plan, which is the Standard Plan. Once a plan is chosen, the member has a 30-day grace period in which to switch to the other plan. After the grace period is over, the member is not allowed to make any further changes. Starting from January 2016, however, CPF members are allowed to make the plan choice at age 65.

[5] Each plan has two AFs. We shall elaborate on the contributions made to the AFs under the two LIFE plans later.

balance in the RA is zero and the payouts received are more than the premium contributions at the time of the member's death, no bequest is payable.

In the LIFE scheme, two annuity premium contributions, one at age 55 and the other at age $64\frac{11}{12}$, are transferred from the RA to the AFs. However, the contributions under the Standard Plan are much larger than those under the Basic Plan. When the member reaches the DDA, the AFs are used to pay for the monthly payouts under the Standard Plan. Under the Basic Plan, the monthly payouts are paid from the RA for the first 25 years (i.e., up to age 90) and then from the AFs subsequently for life.

To help CPF members choose between the two plans, the CPF Board provides some illustrative amounts of payout and bequest under different scenarios of RA contributions.[6] In comparison, the Standard Plan provides higher monthly payouts and lower bequests versus the Basic Plan, given the same amounts of fund contribution. In what follows, we delineate the calculation of the payouts and bequests under the two plans for various combinations of contributions at ages 55 and 65.

3. Actuarial Models of the CPF LIFE Plans

We now provide an actuarial framework to analyze the LIFE plans. The actuarial equivalence principle is adopted to derive the payouts and hence the bequest at death age. We summarize our results for the determination of the payouts, with the mathematical details given in Appendix A.

3.1. *The standard plan*

If a CPF member chooses the default Standard Plan at age 55, a premium of amount C_{S1} will be paid to an annuity fund, which will be called AF1. One month before the member reaches age 65, all savings in the RA, including any new contributions to the account, will be transferred to a second annuity fund, which will be called AF2. We denote this second premium by C_{S2}.[7]

Life-long monthly payouts of P_{S1} and possible bequests of B_{S1_g} at age g are funded by C_{S1} in AF1, while life-long monthly payouts of P_{S2} and possible bequests of B_{S2_g} at age g are funded by C_{S2} in AF2. Thus, the cash outflows from the two AFs consist of the monthly payouts $P_S = P_{S1} + P_{S2}$, starting at age 65 subject to survival, and the amount of bequest $B_{ST_g} = B_{S1_g} + B_{S2_g}$ payable upon death at age g. Figure 1 illustrates the cash flows of the two AFs under the Standard Plan.

Let R_g be the balance in the member's RA at age g. The bequest is then given by

$$B_{ST_g} = \begin{cases} C_{S1} + R_g & \text{if } 55 < g < 65, \\ B_{S1_g} + B_{S2_g} & \text{if } 65 \le g \le \omega, \end{cases}$$

[6] CPF Board provides the LIFE Payout Estimator on its official web site. Members enter their personal information, such as age, gender, RA balances, etc., to obtain estimates of their payouts and bequests. The Estimator, however, does not cater for the input of new money at age 65.

[7] The actual amounts of C_{S1} and C_{S2} depend on the member's RA holdings and are subject to some minimum and maximum values set by the CPF Board.

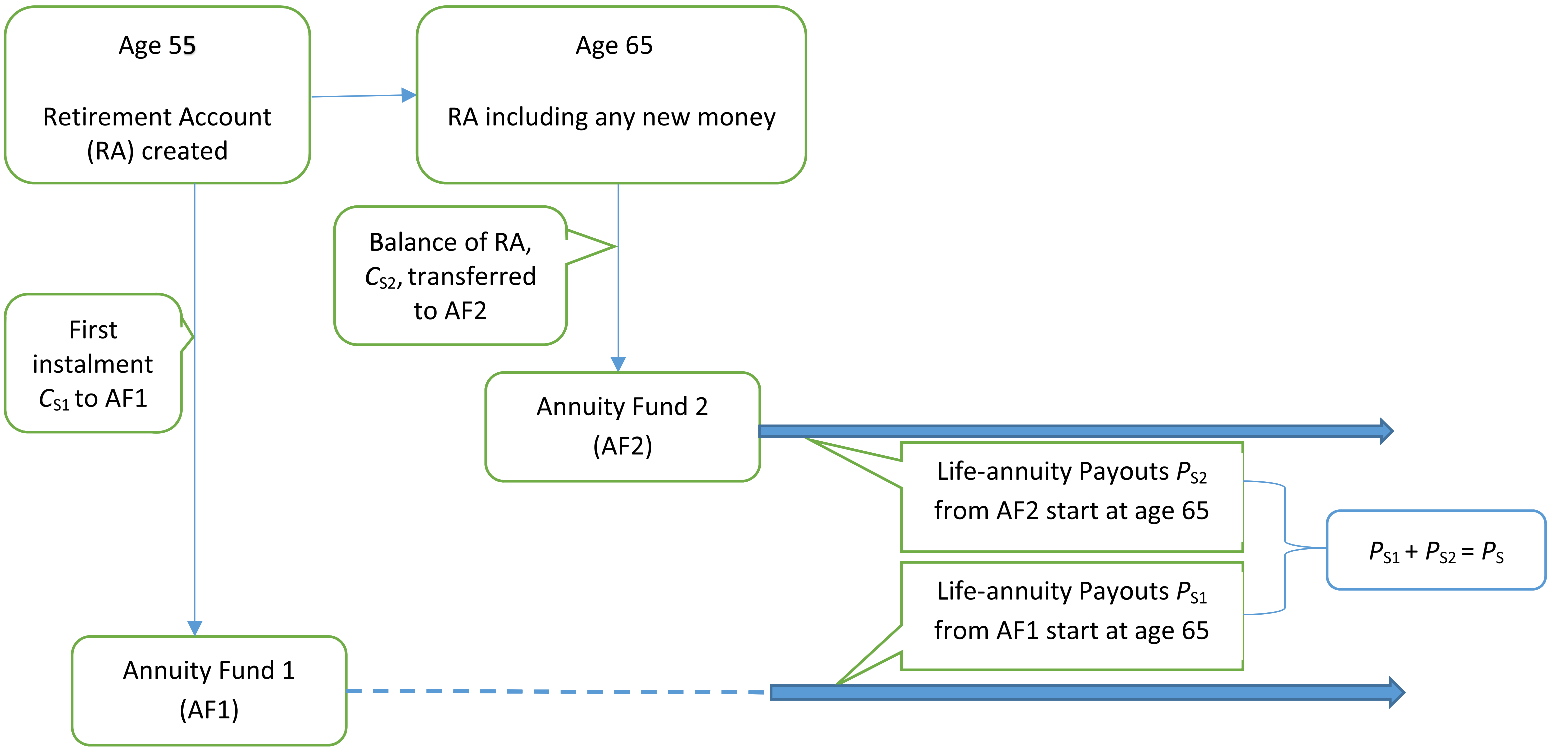

Figure 1. Cash Flows of Annuity Funds in the Standard Plan

where ω is the assumed maximum attainable age, with

$$B_{S1_g} = \max\left[C_{S1} - \left(g - 64\frac{11}{12}\right) \times 12P_{S1},\ 0\right]$$

and

$$B_{S2_g} = \max\left[C_{S2} - \left(g - 64\frac{11}{12}\right) \times 12P_{S2},\ 0\right].$$

Note that if the member dies before reaching age 65, there is no second annuity premium contribution and the bequest is equal to the amount of the first annuity premium C_{S1} plus the balance in the RA at the time of death. As shown in Appendix A, the payouts P_{S1} and P_{S2} can obtained as[8]

$$P_{S1} = \frac{C_{S1} \times (1 - A^{\,1\ \ (12)}_{55:\,\overline{10+t_1}|})}{12 \times {}_{10}E_{55} \times [\ddot{a}^{(12)}_{65} - (I^{(12)}A)^{\,1\ \ (12)}_{65:\,\overline{t_1}|}]}$$

and

$$P_{S2} = \frac{C_{S2} \times (1 - A^{\,1\ \ (12)}_{65:\,\overline{t_2}|})}{12 \times [\ddot{a}^{(12)}_{65} - (I^{(12)}A)^{\,1\ \ (12)}_{65:\,\overline{t_2}|}]}.$$

The monthly payouts to the annuitant are made up of the sum of the above quantities. Thus, the payouts for the Standard Plan are relatively easy to compute.

3.2. *The basic plan*

If the member chooses the Basic Plan at age 55, two annuity premium payments, one at age 55 of amount C_{B1} and one at age 65 of amount C_{B2}, are paid to AF1 and AF2, respectively.[9] The cash outflows of the plan consist of the monthly payouts P_B starting at age 65 and possible bequest B_{BT_g} payable upon death at age g. In particular, P_B are funded by the RA from age 65 to 90. After age 90, two separate monthly payouts, P_{B1} and P_{B2} with $P_{B1} + P_{B2} = P_B$, are funded by AF1 and AF2, respectively. Figure 2 illustrates the cash flows of the two AFs in addition to the annuity-certain component under the Basic Plan.

The bequest at age of death g, B_{BT_g}, is given by

$$B_{BT_g} = \begin{cases} C_{B1} + R_g & \text{if } 55 < g < 65, \\ C_{B1} + C_{B2} + R_g & \text{if } 65 \le g \le 90, \\ B_{B1_g} + B_{B2_g} & \text{if } 90 < g \le \omega, \end{cases}$$

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[8] Standard notations in actuarial mathematics, such as $\ddot{a}^{(m)}_{\overline{t}|}$, $\ddot{a}^{(m)}_x$, $A^{\,1\ \ (m)}_{x:\,\overline{t}|}$ and $(I^{(m)}A)^{\,1\ \ (m)}_{x:\,\overline{t}|}$ are not defined here. Readers may refer to Bowers *et al.* (1997) for their definitions. See Appendix A for the definitions of t_1 and t_2.

[9] C_{B1} is determined at age 55 given the member's RA balance, subject to some limits set by the CPF Board. The CPF Board provides an estimate that C_{B1} is about 10% of R_{55} and C_{B2} is about 10% of any new contribution to the RA at age 65. If there is no new contribution, $C_{B2} = 0$.

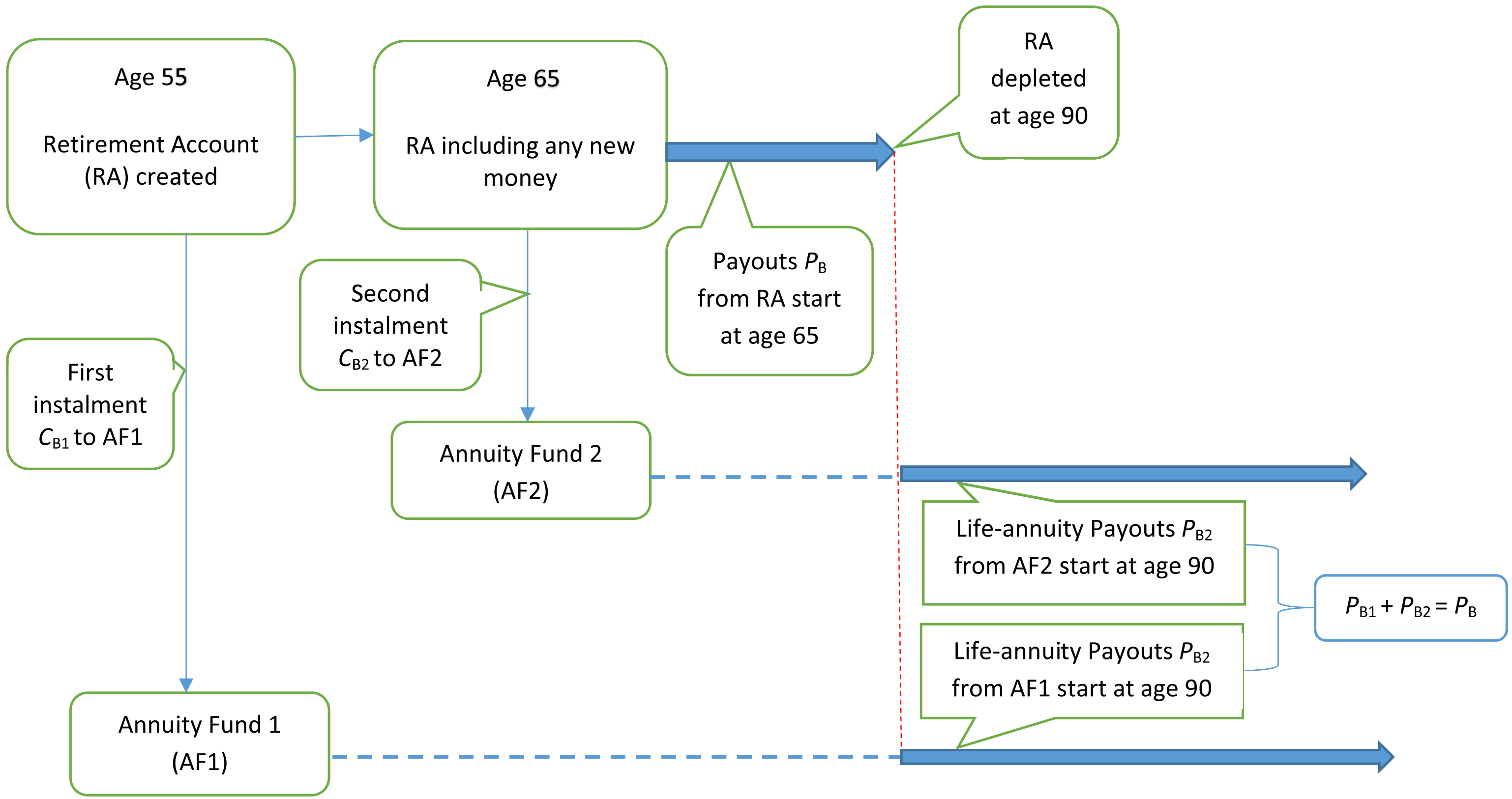

Figure 2. Cash Flows of Annuity Funds in the Basic Plan

where

$$B_{B1_g} = \max[C_{B1} - (g - 90) \times 12P_{B1},\ 0]$$

and

$$B_{B2_g} = \max[C_{B2} - (g - 90) \times 12P_{B2},\ 0].$$

As the RA is used to fund the monthly payouts between age 65 and 90, R_g gradually diminishes during that period until it is depleted upon the member reaching age 90. Thus, the Basic Plan is composed of two annuity components: an annuity-certain and a whole-life annuity.[10]

As members are to receive stable payouts seamlessly in the two phases of the payouts (before and after age 90), the amounts C_{B1} and C_{B2} have to be determined under the constraint of a level annuity throughout. In Appendix A, we derive the joint determination of the contributions and the payouts.[11] The monthly payouts P_{B1} and P_{B2} and the required premium C_{B1} and C_{B2} can be obtained by solving[12]

$$P_{B1} = \frac{C_{B1} \times (1 - A^{1}_{55:\ \overline{35+t'_1}|}{}^{(12)})}{12 \times {}_{35}E_{55} \times [\ddot{a}^{(12)}_{90} - (I^{(12)}A)^{1}_{90:\ \overline{t'_1}|}{}^{(12)}]}$$

and

$$P_{B2} = \frac{C_{B2} \times (1 - A^{1}_{65:\ \overline{25+t'_2}|}{}^{(12)})}{12 \times {}_{25}E_{65} \times [\ddot{a}^{(12)}_{90} - (I^{(12)}A)^{1}_{90:\ \overline{t'_2}|}{}^{(12)}]}$$

subject to two annuity-certain constraints involving C_{B1} and C_{B2}. Unlike the Standard Plan, however, the payouts of the Standard Plan have to be numerically determined through iteration.

4. Some Illustrative Plan Outcomes

To compute the payouts and bequests of the LIFE plans, we need to specify the annuity fund rate of return and mortality assumptions. As the CPF Board is currently crediting 4% interest for the RA savings, we assume the investment return rate to be fixed at 4% per annum.[13] For the mortality rates, we note that Singapore does not have a mortality table for

[10] We calculate the monthly payouts from ages 65 to 90 based on the methodology of an annuity-certain. Upon death of the annuitant before age 90, the balance of the RA is returned to the beneficiaries. In contrast, AF1 and AF2 fund the whole-life annuity after age 90.

[11] This is in contrast to the Standard Plan, for which the payouts can be determined conditional on the contributions.

[12] See Appendix A for the definitions of t'_1 and t'_2.

[13] CPF funds are invested in a portfolio of Singapore Government Securities paying fixed coupon rates over a long period with a government guaranteed floor rate of 4% per annum. The CPE scheme also pays additional 1% interest to some portions of the RA savings. This feature can be easily incorporated in our model.

annuitants. If the Singapore Complete Life Tables are used, the mortality is likely to be overstated.[14] To circumvent the problem of lack of annuitant mortality tables in Singapore, we use the U.S. RP2014 annuitant mortality tables (denoted as RP) as a proxy for the survival distributions of the CPF LIFE members.[15] Finally, we use the constant force-of-mortality assumption within each year of age in our calculations.

For illustration purpose, we consider three cases of contributions to the RA, each with a total of 150,000, for both male and female annuitants: (1) 150,000 at age 55 with no contribution at age 65, (2) 100,000 at age 55 and 50,000 at age 65, and (3) 75,000 each at ages 55 and 65.[16] The amounts of the payouts and bequests at different scenarios of age at death are presented in Table 1. To understand the outcomes, consider a female member with a contribution of 150,000 at age 55 and none at age 65 (second column of the table). The Standard Plan provides her with monthly payouts of 1258 starting at age 65, comprising of 668 and 590 from funds AF1 and AF2, respectively. The projected amounts of bequest decrease from 183,559 for age at death of 65 to 499 for age at death of 80. The premiums paid from the RA to the funds AF1 and AF2 are 77,500 and 107,318, respectively. The term of the decreasing term insurance under AF1 is 9 years and 7 months while that under AF2 is 15 years and 1 month.

Some results from Table 1 can be observed. First, payouts for male members are higher than those for female members in both plans, as male members have higher mortality rates than female members. However, although the payouts for male members are about 5.8% higher than those for female members for the Standard Plan, this difference is reduced to less than 2% for the Basic Plan. The higher percentage difference between the genders for the Standard Plan is due to the larger weight of the whole-life annuity component for the Standard Plan. Second, with more funds contributed to the AFs and lower possible bequests for the Standard Plan, the payouts in the Standard Plan are about 15% and 20% more than those in the Basic Plan for female and male members, respectively, over all the cases considered. Third, as the Standard Plan does not have RA savings after age 65 and has higher payouts than the Basic Plan, its bequests are generally lower than those of the

[14] This is due to the fact that only CPF members with RA balance of more than 40,000 are required to join the scheme. Hence, low-income retirees with possibly higher mortality rates are likely to be excluded from the scheme. As the poor are expected to have higher mortality rates than the rich (see Attanasi and Hoynes (2000) for a detailed discussion of the relationship between mortality rates and wealth), this may cause the mortality rates of the CPF members to be lower than those of the general population. Furthermore, although the LIFE scheme is mandatory, CPF members with poor medical conditions can be exempted from the scheme. This exemption will further lower the mortality rates of members in the scheme.

[15] See *RP-2014 Mortality Tables Report* (Society of Actuaries (2014a)) for the methodology of the construction of these mortality tables. This proxy, as well as others used in this paper, are selected for ease of reference to enhance result replications. It is not our intention to argue that these tables are the most suitable for the Singapore annuitant population. As pointed out by Doyle *et al.* (2004), many countries have not collected enough data to derive annuitant cohort tables. In such a circumstance, insurers frequently make use of annuitant cohort tables from other countries having extensive annuity markets, and then they transform them to approximate their own national experience. This approach has been adopted in some studies on annuity markets of Singapore and Australia, see, for example, Fong (2002) and Doyle *et al.* (2004). Another advantage of using the RP-2014 Mortality Tables is that mortality improvement factors can be easily incorporated for the sensitivity analysis of the mortality assumption. See the next section for further results.

[16] All monetary amounts in this paper are in Singapore dollar. For simplicity, the dollar signs are suppressed. These samples of contributions are for illustration only, and have no implications for income adequacy. For discussion of the issue of adequacy, see Chia and Tsui (2003).

Table 1. Illustrative Examples of LIFE Plan Outcomes

	Female			Male		
	$C = 150{,}000$ $N = 0$	$C = 100{,}000$ $N = 50{,}000$	$C = 75{,}000$ $N = 75{,}000$	$C = 150{,}000$ $N = 0$	$C = 100{,}000$ $N = 50{,}000$	$C = 75{,}000$ $N = 75{,}000$
Standard Plan						
Payout P_{S1}	668	668	647	711	711	688
Payout P_{S2}	590	458	412	618	480	432
Total Payout P_S	1258	1126	1059	1329	1191	1120
Bequest at age 65	183,559	159,679	148,941	183,488	159,615	148,880
Bequest at age 70	108,057	92,100	85,395	103,730	88,155	81,668
Bequest at age 75	35,908	27,874	25,095	32,519	25,243	22,726
Bequest at age 80	499	387	349	0	0	0
Bequest at age 85	0	0	0	0	0	0
Premium C_{S1}	77,500	77,500	75,000	77,500	77,500	75,000
Premium C_{S2}	107,318	83,305	75,000	107,318	83,305	75,000
t_1 (years)	$9\frac{7}{12}$	$9\frac{7}{12}$	$9\frac{7}{12}$	9	9	9
t_2 (years)	$15\frac{1}{12}$	$15\frac{1}{12}$	$15\frac{1}{12}$	$14\frac{5}{12}$	$14\frac{5}{12}$	$14\frac{5}{12}$
Basic Plan						
Payout P_{B1}	1095	730	547	1114	742	557
Payout P_{B2}	0	245	367	0	249	373
Total Payout P_B	1095	975	914	1114	991	930
Bequest at age 65	217,619	194,987	183,671	218,776	195,757	184,247
Bequest at age 70	190,258	170,615	160,794	190,947	170,963	160,971
Bequest at age 75	156,969	140,963	132,960	157,089	140,798	132,652
Bequest at age 80	116,468	104,887	99,097	115,895	104,097	98,198
Bequest at age 85	67,193	60,995	57,896	65,777	59,445	56,280
Premium C_{B1}	8340	5560	4170	5917	3945	2959
Premium C_{B2}	0	3257	4886	0	2420	3630
t'_1 (years)	$\frac{7}{12}$	$\frac{7}{12}$	$\frac{7}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{5}{12}$
t'_2 (years)	0	$1\frac{1}{12}$	$1\frac{1}{12}$	0	$\frac{9}{12}$	$\frac{9}{12}$

Notes: C = Amount transferred to the Retirement Account at age 55. N = New money contribution at age 65. The mortality tables used are RP2014, and the interest rate assumed is 4%. P_{S1} and P_{S2} are the monthly payouts from the annuity funds AF1 and AF2, respectively, of the Standard Plan, and $P_S = P_{S1} + P_{S2}$. P_{B1} and P_{B2} are the monthly payouts from the annuity funds AF1 and AF2, respectively, of the Basic Plan, and $P_B = P_{B1} + P_{B2}$. C_{S1} and C_{S2} are the premiums contributed to the annuity funds AF1 and AF2, respectively, of the Standard Plan. C_{B1} and C_{B2} are defined similarly for the Basic Plan. t_1 and t_2 are the terms (in years) of the decreasing term insurance coverage provided by the annuity funds AF1 and AF2, respectively, of the Standard Plan. t'_1 and t'_2 are defined similarly for the Basic Plan.

Basic Plan. In particular, there is no bequest at age 85 for the Standard Plan over all cases considered. For the Basic Plan, however, a small bequest is still payable at age 85.

5. Sensitivity Analysis of the Model Assumptions

As a government statutory authority, the CPF Board is the sole administrator of the LIFE scheme. This provides substantial economies of scale to lower the operating costs.

However, unlike a private life-annuity provider, the CPF Board does not take on any risks as the administrator. As a result, CPF members must bear and share the inflation risk, interest rate risk and longevity risk.

To sustain the long-term viability of the funding of the LIFE scheme, the monthly payouts of the scheme must be subject to changes from time to time. The changes will depend on the realized mortality rate and investment return rate. In particular, if the experienced mortality rate is lower than the model assumption and/or the actual investment return is less than the expected, the monthly payouts have to be adjusted downwards.

To examine the sensitivity of the monthly payouts to the model assumptions for each plan, we consider alternative model assumptions and parameters. We study how changes in the interest rate and the mortality rate affect the monthly payouts. For the interest rate parameter, we consider a drop of the credited interest rate from 4% to 3.5% and 3%.[17] For mortality variation, we modify the RP2014 mortality tables by incorporating the MP2014 mortality improvement factors for the cohort aged 55 in 2014.[18] The MP2014 mortality tables with a long-term mortality improvement rate of 1%, denoted as MPA, is considered. This improvement reflects the general historical age/period and cohort effects according to

Table 2. Monthly Payouts of the Standard Plan under Different Model Assumptions

	Female			Male		
	RP	MPA	MPB	RP	MPA	MPB
Interest rate = 4%						
$C = 150{,}000$, $N = 0$	1258	1183	1163	1329	1245	1222
$C = 100{,}000$, $N = 50{,}000$	1126	1058	1041	1191	1115	1094
$C = 75{,}000$, $N = 75{,}000$	1059	995	978	1120	1048	1029
Interest rate = 3.5%						
$C = 150{,}000$, $N = 0$	1132	1060	1042	1198	1117	1096
$C = 100{,}000$, $N = 50{,}000$	1025	960	943	1086	1013	993
$C = 75{,}000$, $N = 75{,}000$	971	909	893	1029	959	940
Interest rate = 3%						
$C = 150{,}000$, $N = 0$	1014	946	929	1075	999	979
$C = 100{,}000$, $N = 50{,}000$	930	867	852	987	917	898
$C = 75{,}000$, $N = 75{,}000$	887	827	812	941	874	857

Notes: C = Amount transferred to the Retirement Account at age 55. N = New money contribution at age 65. RP = U.S. RP2014 annuitant mortality tables. MPA and MPB are the U.S. MP 2014 mortality tables with long-term mortality improvement rates of 1% and 1.5%, respectively.

[17] The credited interest is dependent on the returns of the government's investments. The rate of 4% has been maintained since the start of the LIFE program.

[18] See *Mortality Improvement Scale MP-2014 Report* (Society of Actuaries (2014b)) for a description of the mortality improvement estimation methodology.

Table 3. Monthly Payouts of the Basic Plan under Different Model Assumptions

	Female			Male		
	RP	MPA	MPB	RP	MPA	MPB
Interest rate $= 4\%$						
$C = 150{,}000$, $N = 0$	1095	1063	1056	1114	1085	1077
$C = 100{,}000$, $N = 50{,}000$	975	946	939	991	965	958
$C = 75{,}000$, $N = 75{,}000$	914	887	881	930	905	899
Interest rate $= 3.5\%$						
$C = 150{,}000$, $N = 0$	984	952	945	1003	974	966
$C = 100{,}000$, $N = 50{,}000$	887	857	851	904	877	870
$C = 75{,}000$, $N = 75{,}000$	838	810	804	854	828	822
Interest rate $= 3\%$						
$C = 150{,}000$, $N = 0$	882	850	843	901	871	864
$C = 100{,}000$, $N = 50{,}000$	804	775	769	822	794	788
$C = 75{,}000$, $N = 75{,}000$	766	737	731	782	756	749

Notes: C = Amount transferred to the Retirement Account at age 55. N = New money contribution at age 65. RP = U.S. RP2014 annuitant mortality tables. MPA and MPB are the U.S. MP 2014 mortality tables with long-term mortality improvement rates of 1% and 1.5%, respectively.

experts' opinions on the long-term mortality patterns. Furthermore, we also consider a long-term mortality improvement rate of 1.5% and denote this case as MPB.

Tables 2 and 3 provide the monthly payouts of the three illustrative cases of fund contributions considered in Section 4. For both plans the drop in the monthly payouts from the case of 4% interest to 3% interest is about 17% to 20%, and the drop appears to be approximately linear in the range of interest rates considered. The amount of the percentage drop in the payouts is quite similar across different genders, plan choices and mortality assumptions. With improvement in mortality from RP to MPA, the payouts drop by about 6% to 7% for the Standard Plan and for about 2% to 4% for the Basic Plan. On the other hand, with improvement in mortality from RP to MPB, the payouts drop by about 7% to 9% for the Standard Plan and for about 3% to 5% for the Basic Plan. As the Basic Plan has larger weight for the annuity-certain component, which does not depend on the mortality assumption, the results of less payouts drop for the Basic Plan due to mortality improvement are perhaps not surprising. Overall, our results show that the interest credited to the fund is an important factor in determining the payouts, and this factor has higher impact on the payouts than the mortality experience in both plans.

6. Some Possible Plan Modifications

The LIFE scheme provides level annuity payments with no increments to meet any possible future rising costs of living. This exposes members to inflation risk, which diminishes the real values of the annuity payments (Bodie and Pesando (1983)). There are many strategies in dealing with inflation risk (Brown *et al.* (2001)). One strategy is the use of real annuities for which periodic payments are indexed to annual changes in a benchmark price index. In many developed countries, the availability of inflation-indexed government bonds and securities (Deacon *et al.* (2004)) makes it feasible for annuity providers to offer protection against inflation. However, inflation-indexed government securities are not available in Singapore.

To alleviate the adverse effect of inflation on annuitants, two possible modifications of the LIFE plans can be considered. First, we may consider monthly payouts with smaller initial amounts followed by subsequent annual step-ups. The second modification is to consider a deferment feature (Blake (1999)) of the annuity payments by raising the Draw Down Age.[19] As the retirement age in Singapore will gradually increase and mortality improvement is expected, raising the DDA may be a viable option.

Our actuarial models can be used to evaluate the effects of the above plan modifications on the payouts. Table 4 summarizes the results for different values of DDA (65, 66, 67, 68) and annual step-up rate r (0, 1%, 2%) on the initial amount of payout with the contribution to the RA at age 55 being 100,000 and 150,000, under MPA mortality assumption with $i = 4\%$.

The following results can be observed from the table. First, without the step-up, the level monthly payout increases by 6.5% and 7% for every one-year deferment of the DDA for female and male members, respectively, under the Standard Plan. However, the level monthly payout increases by about 6% per one-year deferment of the DDA for both genders under the Basic Plan. Second, for every 1% point increase in the step-up rate r

Table 4. Initial Payouts with Different Step-up Rates and Deferred Draw Down Age

	Standard Plan							
	Female				Male			
DDA	65	66	67	68	65	66	67	68
$C = 100{,}000$								
$r = 0\%$	799	851	908	970	844	902	966	1035
$r = 1\%$	714	763	816	874	759	814	873	939
$r = 2\%$	633	679	729	784	678	730	786	847
$C = 150{,}000$								
$r = 0\%$	1183	1260	1344	1435	1245	1330	1423	1525
$r = 1\%$	1056	1128	1207	1293	1119	1199	1286	1382
$r = 2\%$	936	1004	1077	1158	999	1074	1156	1247

[19] From January 2016 onward, CPF members are given the flexibility to defer the payout start age from the DDA of 65 to up to 70, and are only required to choose their LIFE plans at the time when they wish to start payouts (Singapore CPF Board (2015)).

Table 4. (*Continued*)

	Basic Plan							
	Female				Male			
DDA	65	66	67	68	65	66	67	68
$C = 100{,}000$								
$r = 0\%$	709	751	797	846	723	767	815	867
$r = 1\%$	629	669	712	759	645	686	732	781
$r = 2\%$	555	592	633	677	571	611	653	700
$C = 150{,}000$								
$r = 0\%$	1063	1127	1195	1270	1085	1151	1223	1301
$r = 1\%$	944	1003	1068	1138	967	1029	1097	1172
$r = 2\%$	832	888	949	1015	857	916	980	1050

Notes: C = Amount transferred to the Retirement Account at age 55, DDA = Draw Down Age, r = annual rate of step-up. Mortality tables used are MPA and interest rate is 4%.

with a given DDA, the initial payout drops by about 10% in both plans. For example, with a 2% step-up the initial monthly payout is about 80% of the level monthly payout and it takes about 11 years for the step-up payouts to reach the level payouts without step-up. Third, for a Standard Plan with three-year deferment and 2% step-up, the initial payout is almost the same as the case with level monthly payouts without DDA deferment for both genders. In contrast, for a Basic Plan with three-year deferment and 2% step-up, the initial payout is about 95% to 97% of the case with level monthly payouts without deferment for both genders. Thus, the combination with 2% step-up and three-year deferment may be an attractive and viable choice for members who wish to address the effects of inflation on their retirement income.

7. Summary and Future Studies

We have constructed an actuarial framework to analyze the CPF LIFE scheme. The monthly payouts of the Standard and Basic Plans are derived based on the actuarial equivalence principle. We examine the impacts on the monthly payouts due to possible parameter changes in the scheme, in particular, changes in mortality assumption and the credited rate of interest.

Our models can be used to study other modifications and enhancements of the LIFE program. First, without any underwriting process individuals do not need to disclose their health status. It is important to investigate how adverse selection may affect the cashflows of the annuity funds in each plan. Second, management of the funds in the two plans and the longevity risks of different generation cohorts are important issues to be considered by the scheme administrator. Our actuarial framework provides a useful platform to investigate these issues. Third, in the process of implementation the amount of monthly payouts may be adjusted yearly, depending on the actual investment outcomes and mortality experiences. To reduce the effects of investment and mortality risks, a buffer fund may be set up. Our actuarial models may be applied to study the set-up of such buffer funds. Fourth, our framework can be used to calculate the adjusted payout if the annuitants are allowed to switch their plan after joining the scheme. Research on these issues are ongoing.

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Appendix A. Determination of Plan Payout and Bequest

In this appendix, we provide the mathematical details of the determination of the payouts and bequests of the LIFE plans. As described in Section 3, the LIFE plans operate with two life funds and they provide level payouts throughout the life of the annuitant (payouts, however, may subsequently vary, subject to possible future experience refinement). These conditions present challenges to the determination of the actuarially fair payouts. In what follows we derive algorithms to calculate these payouts, which require iterative computation.

We first consider the Standard Plan. Let the annual annuity fund rate of return be i, and let v be the discount factor (i.e., $v = 1/(1+i)$). Denote the monthly curtate future lifetime random variables by $K_{55}^{(12)}$ and $K_{65}^{(12)}$, which are the future lifetimes of a CPF member at ages 55 and 65, respectively, rounded to the lower month of a year. We further denote the present values of the future loss random variables from the two premium payments C_{S1} and C_{S2} by L_{S1} and L_{S2}, respectively.

From the results in Section 3.1, we have

$$L_{S1} = \begin{cases} -C_{S1} + C_{S1} \times v^{K_{55}^{(12)}+\frac{1}{12}} & \text{for } K_{55}^{(12)} = 0, \dfrac{1}{12}, \ldots, 9\dfrac{11}{12} \\ -C_{S1} + v^{10} \times 12P_{S1} \times \ddot{a}^{(12)}_{\overline{K_{55}^{(12)}-9\frac{11}{12}}|} & \\ +\max[C_{S1} - P_{S1} \times (12K_{55}^{(12)} - 119),\ 0] \times v^{K_{55}^{(12)}+\frac{1}{12}} & \text{for } K_{55}^{(12)} = 10, 10\dfrac{1}{12}, \ldots \end{cases}$$

and

$$L_{S2} = -C_{S2} + 12P_{S2} \times \ddot{a}^{(12)}_{\overline{K^{(12)}_{65}+\frac{1}{12}}|} + \max[C_{S2} - P_{S2} \times (12K^{(12)}_{65} + 1),\ 0] \times v^{K^{(12)}_{65}+\frac{1}{12}}$$

for $K^{(12)}_{65} = 0, \frac{1}{12}, \ldots$. Let t_1 and t_2 be the terms (in years) of the monthly decreasing term insurance covered under the premiums C_{S1} and C_{S2}, respectively, subject to[20]

$$12t_1 P_{S1} < C_{S1} < (12t_1 + 1)P_{S1}$$

and

$$12t_2 P_{S2} < C_{S2} < (12t_2 + 1)P_{S2}$$

Based on the actuarial equivalence principle, with the actuarial present value of cash inflows equated to the actuarial present value of cash outflows, the monthly payouts under the Standard Plan are obtained as

$$P_{S1} = \frac{C_{S1} \times (1 - A^{1\ (12)}_{55:\ \overline{10+t_1}|})}{12 \times {}_{10}E_{55} \times [\ddot{a}^{(12)}_{65} - (I^{(12)}A)^{1\ (12)}_{65:\ \overline{t_1}|}]}$$

and

$$P_{S2} = \frac{C_{S2} \times (1 - A^{1\ (12)}_{65:\ \overline{t_2}|})}{12 \times [\ddot{a}^{(12)}_{65} - (I^{(12)}A)^{1\ (12)}_{65:\ \overline{t_2}|}]}$$

For the Basic Plan, we denote the present values of the future losses of the two AFs with single premiums C_{B1} and C_{B2} by L_{B1} and L_{B2}, respectively, which are given by

$$L_{B1} = \begin{cases} -C_{B1} + C_{B1} \times v^{K^{(12)}_{55}+\frac{1}{12}} & \text{if } K^{(12)}_{55} = 0, \dfrac{1}{12}, \ldots, 34\dfrac{11}{12} \\ -C_{B1} + v^{35} \times 12P_{B1} \times \ddot{a}^{12}_{\overline{K^{(12)}_{55}-34\frac{11}{12}}|} & \\ + \max[C_{B1} - P_{B1} \times (12K^{(12)}_{55} - 419),\ 0] \times v^{K^{(12)}_{55}+\frac{1}{12}} & \text{if } K^{(12)}_{55} = 35, 35\dfrac{1}{12}, \ldots \end{cases}$$

and

$$L_{B2} = \begin{cases} -C_{B2} + C_{B2} \times v^{K^{(12)}_{65}+\frac{1}{12}} & \text{if } K^{(12)}_{65} = 0, \dfrac{1}{12}, \ldots, 24\dfrac{11}{12} \\ -C_{B2} + v^{25} \times 12P_{B2} \times \ddot{a}^{12}_{\overline{K^{(12)}_{65}-24\frac{11}{12}}|} & \\ + \max[C_{B2} - P_{B2} \times (12K^{(12)}_{65} - 299),\ 0] \times v^{K^{(12)}_{65}+\frac{1}{12}} & \text{if } K^{(12)}_{65} = 25, 25\dfrac{1}{12}, \ldots \end{cases}$$

[20] Note that the terms of these insurances, t_1 and t_2, respectively, are invariant to C_{S1} and C_{S2}.

Let t'_1 and t'_2 be the terms (in years) of the monthly decreasing term insurance covered by the single premiums C_{B1} and C_{B2}, respectively, subject to[21]

$$12t'_1 P_{B1} < C_{B1} < (12t'_1 + 1)P_{B1}$$

and

$$12t'_2 P_{B2} < C_{B2} < (12t'_2 + 1)P_{B2}$$

Invoking the actuarial equivalence principle, the monthly payouts P_{B1} and P_{B2} and the required premium C_{B1} and C_{B2} can be obtained by solving

$$P_{B1} = \frac{C_{B1} \times (1 - A^{1\ \ (12)}_{55:\ \overline{35+t'_1}|})}{12 \times {}_{35}E_{55} \times [\ddot{a}^{(12)}_{90} - (I^{(12)}A)^{1\ \ (12)}_{90:\ \overline{t'_1}|}]}$$

subject to the annuity-certain constraint $(1+i)^{10}R_{55} = 12P_{B1} \times \ddot{a}^{12}_{\overline{25}|}$, where R_{55} is the balance of the RA at age 55 after making the premium C_{B1}, and

$$P_{B2} = \frac{C_{B2} \times (1 - A^{1\ \ (12)}_{65:\ \overline{25+t'_2}|})}{12 \times {}_{25}E_{65} \times [\ddot{a}^{(12)}_{90} - (I^{(12)}A)^{1\ \ (12)}_{90:\ \overline{t'_2}|}]}$$

subject to the other annuity-certain constraint $C_{B2} = N - 12P_{B2} \times \ddot{a}^{12}_{\overline{25}|}$, where N is the new contribution to the RA at age 65. The payouts can be determined numerically by iteration.

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[21] Similar to the case of the Standard Plan, these results show that t'_1 and t'_2 are invariant to C_{B1} and C_{B2}, respectively.

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